ASSIGNMENT 2 – SOLUTIONS

WMD651 – Water Management Systems Design

Winter 2021

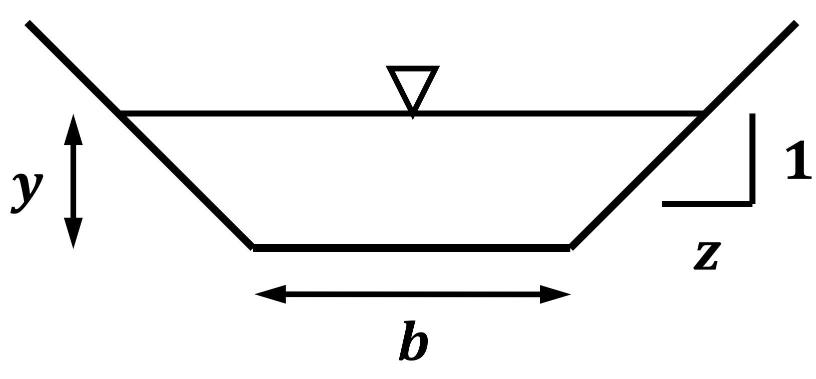
Due Date: Monday February 08, 2021

Instructions:

Below are five questions based on lessons 1 through 6. Solutions may be hand-written or typed. All questions are exercises in communication, so answers should be clear and well-structured. Any unclear answers will not be graded. List any assumptions in addition to those given by a problem. Show all work. The assignment is out of 20 marks.

QUESTION 1 [5 MARKS]

Below is a trapezoidal section for an open channel. The channel base width is b = 5 m, the bed slope is S_0 = 0.005 m/m, the Manning roughness is n = 0.015, and the side slope is characterized by z = 3 m/m. If the channel has a flow of Q = 20 m³/s, determine the channel's normal depth y_n. Perform up to three iterations, and comment on whether and why additional iterations are needed. Note that for a trapezoidal channel section, $A = y(b + zy)$ and $R = \frac{A}{b+2y\sqrt{1+z^2}}$. **Hint:** *see the normal depth example in the lesson 4 slides.*



Solution:

Step 1: Given parameters

- Flow: Q = 20 m³/s

- Manning roughness: n = 0.015
- Bed slope: S_0 = 0.005 m/m
- Channel base width: b = 5 m
- Channel side slope: z = 3 m/m

Step 2: List equations needed to solve the problem

- Manning equation: $Q = \frac{1}{n} A R^{2/3} S_f^{1/2}$
- Uniform flow: $S_f = S_0$

Step 3: Develop a system equation that will need to be solved

- Rearrange the Manning equation to collect known (left) and unknown terms (right:
    - Manning equation: $Q = \frac{1}{n} A R^{2/3} S_0^{0.5}$
    - Of the terms in the Manning equation, we know Q, n, and S_0
    - Unknowns: A and R
- Rearrange to collect known terms on the left hand side: $\frac{nQ}{S_0^{0.5}} = AR^{2/3}$
- $\frac{nQ}{S_0^{0.5}} = AR^{2/3}$ is referred to as the section factor. We must determine the normal depth y_n such that $AR^{2/3} = \frac{nQ}{S_0^{0.5}}$, whereby we can calculate $\frac{nQ}{S_0^{0.5}}$
- Calculate section factor: $\frac{nQ}{S_0^{0.5}} = \frac{0.015(20\ m^3/s)}{(0.005)^{0.5}} = 4.24$ **[1 mark]**

Step 4: Solve the system equation.

- Flow area for a trapezoidal channel: $A = y(b + zy)$
- Hydraulic radius for a trapezoidal channel: $R = \frac{A}{b+2y\sqrt{1+z^2}}$
- We will need to estimate y_n, calculate the section factor, compare the calculated section factor against our target value, and then update and recheck our estimate.
- Trial 1: use y_n = 1 m
    - Area: $A = y(b + zy) = (1\ m)[(5\ m) + (3\ m/m)(1\ m)] = 8\ m^2$
    - Hydraulic radius: $R = \frac{A}{b+2y\sqrt{1+z^2}} = \frac{(8\ m^2)}{(5\ m)+2(1\ m)\sqrt{1+(3\ m/m)^2}} = 0.706\ m$
    - Section factor: $AR^{2/3} = (8\ m^2)(0.706\ m)^{2/3} = 6.34$ **[1 mark]**
    - Our calculated section factor (6.34) is larger than the actual section factor (4.24), so the trial depth (1 m) is too large. **[1 mark for correct evaluation]**
- Trial 2 – use y_n = 0.5 m:
    - Area: $A = y(b + zy) = (0.5\ m)[(5\ m) + (3\ m/m)(0.5\ m)] = 3.25\ m^2$

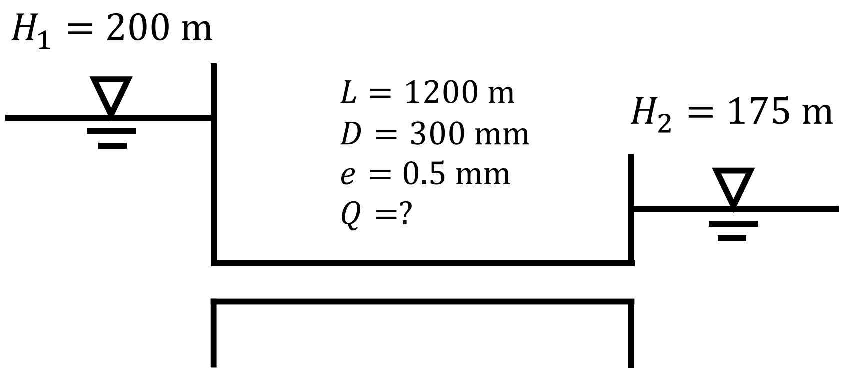
- Hydraulic radius: $R = \frac{A}{b+2y\sqrt{1+z^2}} = \frac{(3.25\ m^2)}{(5\ m)+2(0.5\ m)\sqrt{1+(3\ m/m)^2}} = 0.398\ m$
- Section factor: $AR^{2/3} = (3.25\ m^2)(0.398\ m)^{2/3} = 1.76$
- Our calculated section factor (1.76) is smaller than the actual section factor (4.24), so the trial depth (0.5 m) is too small. The solution is between 0.5 m and 1.0 m. **[1 mark for additional iteration]**

- Trial 3 – test y_n = 0.75 m:
  - Area: $A = y(b+zy) = (0.75\ m)[(5\ m)+(3\ m/m)(0.75\ m)] = 5.44\ m^2$
  - Hydraulic radius: $R = \frac{A}{b+2y\sqrt{1+z^2}} = \frac{(5.44\ m^2)}{(5\ m)+2(0.75\ m)\sqrt{1+(3\ m/m)^2}} = 0.558\ m$
  - Section factor: $AR^{2/3} = (5.44\ m^2)(0.398\ m)^{2/3} = 3.69$
  - Our calculated section factor (3.69) is smaller than the actual section factor (4.24), so the trial depth (0.75 m) is too small. The solution is between 0.75 m and 1.0 m. **[0.5 marks for additional iteration]**

Because our solution has **not** converged yet, additional iterations are needed. **[0.5 marks for stating that additional iterations are needed]** If one were to continue iterating, we would find that the normal depth is y_n = 0.809 m. (You do not need to calculate the exact solution.)

QUESTION 2 [5 MARKS]

Below is a reservoir-pipe-reservoir system with pressurized flow. Determine the system's flow, and perform up to three iterations. Note that you will need to use the Moody chart to determine the friction factor f.



Hint: To solve this problem, begin by developing a system equation, estimating the friction factor f (e.g., start with f = 0.03), calculate the system flow, then determine f from the Moody chart and compare it against your estimate. Perform up to three iterations, and comment on whether further iterations are needed. Your estimated f and calculated f (from the Moody diagram) should gradually get closer and closer with each iteration. See also (i) the supplemental notes at the end of this assignment and (ii) the example in the lesson 3 slides.

Solution:

Step 1: Given parameters.

- Length: L = 1200 m
- Diameter: D = 300 mm = 0.300 m
- Pipe wall roughness: e = 0.5 mm = 0.0005 m
- Upstream reservoir HGL: H_1 = 200 m
- Downstream reservoir HGL: H_2 = 175 m
- Kinematic viscosity: v = 1×10^{-6} m^2/s

Step 2: List equations needed to solve the problem.

- Energy equation: $E_1 + H_{pump} = E_2 + H_{loss}$
- Total head loss: $H_{loss} = H_{friction} + H_{local}$
- Darcy-Weisbach head loss equation: $H_{loss} = f\left(\frac{L}{D}\right)\frac{V^2}{2g}$
- Alternate Darcy-Weisbach: $H_{friction} = K_{friction}Q^2$ where $K_{friction} = fL/2gDA^2$
- Pipe flow area: $A = \frac{1}{4}\pi D^2 = \frac{1}{4}\pi(0.3\ m)^2 = 0.0706\ m^2$
- Relative roughness: $\frac{e}{D} = \frac{0.0005\ m}{0.300\ m} = 1.7 \times 10^{-3}$
- Reynolds number: $Re = \frac{VD}{v} = \frac{QD}{vA}$ (note: v = 1×10^{-6} m^2/s)

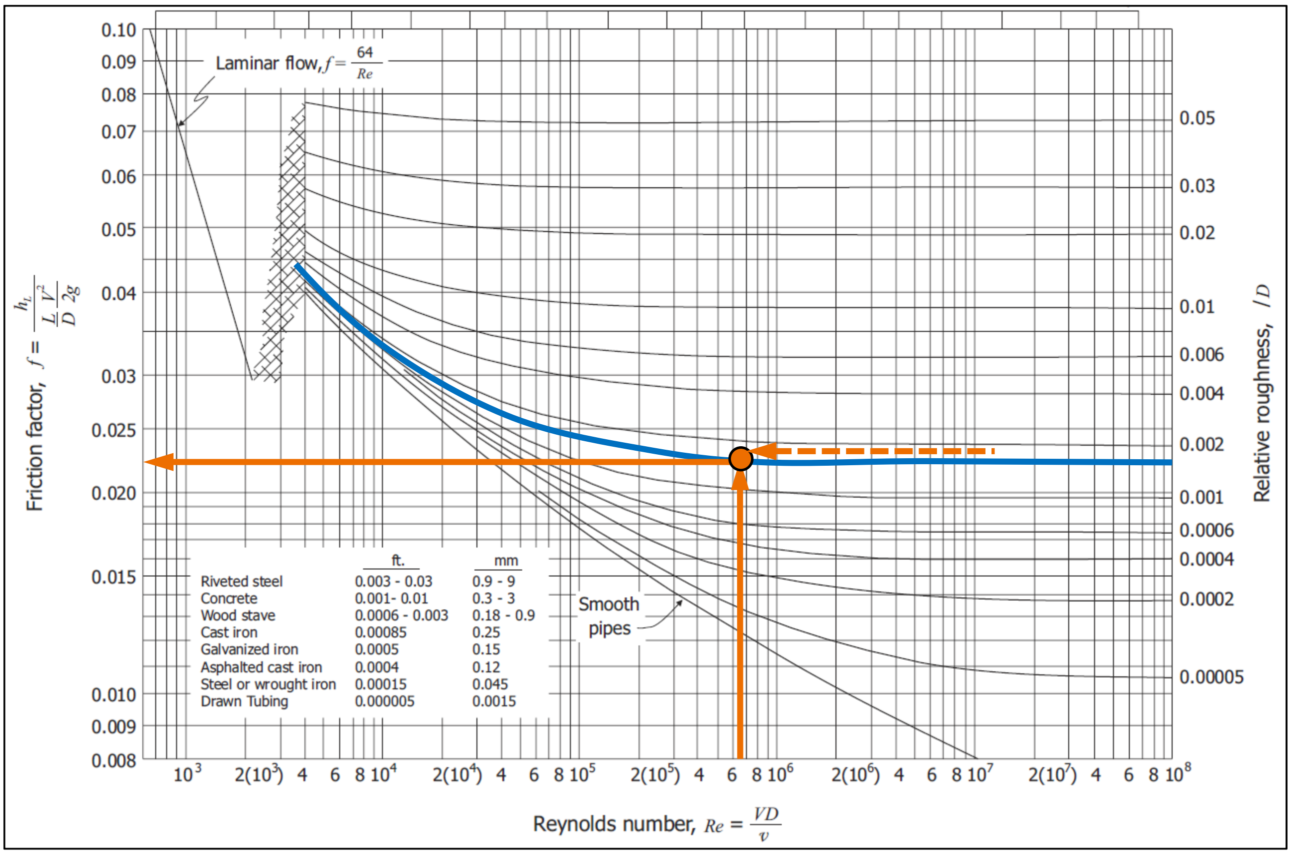
Step 3: Develop a system equation that will need to be solved.

- Energy equation: $E_1 + H_{gain} = E_2 + H_{loss}$
- Point 1 represents the left reservoir, while point 2 represents the right reservoir.
- For reservoirs, $E = H \rightarrow E_1 = H_1$ and $E_2 = H_2$
- There are no pumps in the system ($H_{pump} = 0$)
- Local losses are assumed to be negligible ($H_{local} = 0$)
- Updated energy equation: $H_1 = H_2 + H_{friction}$ **[1 mark for getting to here]**
- Substitute friction loss equation and rearrange: $K_{friction}Q^2 = H_1 - H_2$.
- Rearrange to isolate the flow: $Q = \sqrt{\frac{H_1 - H_2}{K_{friction}}}$ **[1 mark]**

Step 4: Solve the system equation.

- To solve the problem, we need to know $K_{friction}$ and thus f. However, f depends on Q, which we do not yet know. As a result, we will need to estimate f_{trial}, calculate Q, calculate $f_{updated}$ from the Moody diagram, compare f_{trial} against $f_{updated}$, and then update and recheck our estimate.

- Trial 1: use f_{trial} = 0.03
  - Resistance coefficient:
    - $K_{friction} = \frac{fL}{2gDA^2} = \frac{0.03(1200\ m)}{2(9.81\ m/s^2)(0.3\ m)(0.0706\ m^2)^2} = 1224\ s^2/m^5$
  - Flow: $Q = \sqrt{\frac{H_1 - H_2}{K_{friction}}} = \sqrt{\frac{200\ m - 175\ m}{1224\ s^2/m^5}} = 0.143\ m^3/s$ **[1 mark]**
  - Reynolds number: $Re = \frac{QD}{\nu A} = \frac{(0.143\ m^3/s)(0.3\ m)}{(10^{-6}\ m^2/s)(0.0706\ m^2)} = 6.1 \times 10^5$
  - Using $e/D = 1.7 \times 10^{-3}$ (0.0017) and $Re = 6.1 \times 10^5$, we get $f_{updated}$ = 0.0225 from the Moody diagram (see below). This differs from our trial estimate of the friction factor (f_{trial} = 0.03), so another iteration is needed. **[1 mark]**



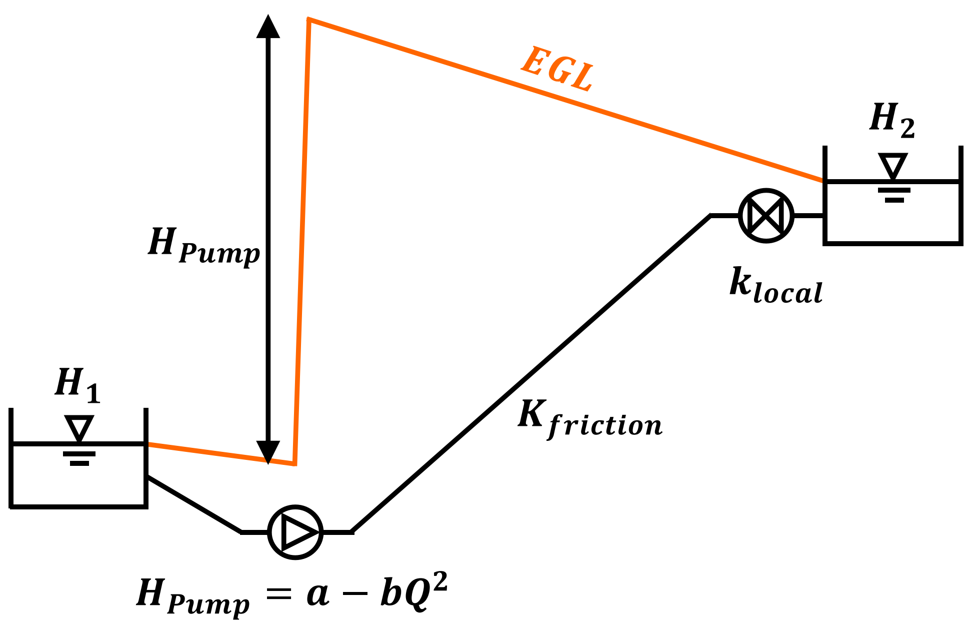
- Trial 2: use f_{trial} = 0.0225
  - Resistance coefficient:
    - $K_{friction} = \frac{fL}{2gDA^2} = \frac{0.0225(1200\ m)}{2(9.81\ m/s^2)(0.3\ m)(0.0706\ m^2)^2} = 918\ s^2/m^5$

- Flow: $Q = \sqrt{\frac{H_1 - H_2}{K_{friction}}} = \sqrt{\frac{200\ m - 175\ m}{918\ s^2/m^5}} = 0.165\ m^3/s$
- Velocity: $V = \frac{Q}{A} = \frac{0.165\ m^3/s}{0.0706\ m^2} = 2.33\ m/s$
- Reynolds number: $Re = \frac{QD}{\nu A} = \frac{(0.165\ m^3/s)(0.3\ m)}{(10^{-6}\ m^2/s)(0.0706\ m^2)} = 7.0 \times 10^5$
- Using $e/D = 1.7 \times 10^{-3}$ (0.0017) and $Re = 7.0 \times 10^5$, we get f_{final} = 0.0225 from the Moody diagram. The updated friction factor is the same as our estimated friction factor, so we have found the solution.

Thus, the system's flow is Q = 0.165 m³/s with a friction factor of f = 0.0225. [1 mark]

QUESTION 3 [5 MARKS]

Below is a pumping pipeline system. The reservoirs have H_1 = 230 m and H_2 = 280 m, and two pump options are being considered.



The TDH curve for pump option #1 has coefficients a = 75 m and b = 180 s²/m⁵. The coefficients for pump option #2 are a = 100 m and b = 270 s²/m⁵. The pipeline has resistance $K_{friction}$ = 90 s²/m⁵ for use with $H_{friction} = K_{friction}Q^2$. Assume local losses are negligible (i.e., K_{local} = 0 s²/m⁵). Complete the following:

a) Determine the system flow if Pump #1 is installed.

Solution:

Step 1: Given parameters

- Reservoir levels: H_1 = 230 m and H_2 = 280 m
- Pump 1: a_1 = 75 m and b_1 = 180 s²/m⁵
- Pump 2: a_2 = 100 m and b_2 = 270 s²/m⁵
- Pipe resistance: $K_{friction}$ = 90 s²/m⁵

Step 2: List equations needed to solve the problem

- Energy equation: $E_1 + H_{pump} = E_2 + H_{loss}$
- Pump TDH: $H_{pump} = a - bQ^2$
- Total head loss: $H_{loss} = H_{friction} + H_{local}$
- Friction head loss: $H_{friction} = K_{friction}Q^n$ (note: n = 2 because our pipe resistance, $K_{friction}$, has units of s²/m⁵)

Step 3: Develop a system equation that will need to be solved

- Start with the energy equation: $E_1 + H_{pump} = E_2 + H_{loss}$
- Point 1 represents the left reservoir, while point 2 represents the right reservoir.
- For reservoirs, $E = H$ → $E_1 = H_1$ and $E_2 = H_2$
- Local losses are assumed negligible ($H_{local} = 0$) → $H_{loss} = H_{friction} = K_{friction}Q^2$
- Update energy equation: $H_1 + (a - bQ^2) = H_2 + K_{friction}Q^2$ **[1 mark]**
  - Note: a and b depend on which pump is used, see further below.
- Rearrange to isolate Q:
  - $H_1 + (a - bQ^2) = H_2 + K_{friction}Q^2$
  - $a - H_2 + H_1 = bQ^2 + K_{friction}Q^2$
  - $a - H_2 + H_1 = (b + K_{friction})Q^2$
  - $Q = \sqrt{\frac{a-(H_2-H_1)}{b+K_{friction}}}$ **[1 mark]**
  - Note that $H_{static} = H_2 - H_1 = 280 - 230 = 50\ m$ → $Q = \sqrt{\frac{a-H_{static}}{b+K_{friction}}}$

Step 4: Solve the system equation

- For pump 1: a_1 = 75 m and b_1 = 180 s²/m⁵
- Flow: $Q = \sqrt{\frac{a-(H_2-H_1)}{b+K_{friction}}} = \sqrt{\frac{(75\ m)-(50\ m)}{(180\ s^2/m^5)+(90\ s^2/m^5)}} = 0.304\ m^3/s$ **[1 mark]**

b) Determine the system flow if Pump #2 is installed.

Solution:

Use the equation derived in part (a) above:

- Pump 2: a_2 = 100 m and b_2 = 270 s²/m⁵
- Flow: $Q = \sqrt{\frac{a-(H_2-H_1)}{b+K_{friction}}} = \sqrt{\frac{(100\ m)-(50\ m)}{(270\ s^2/m^5)+(90\ s^2/m^5)}} = 0.373\ m^3/s$ **[1 mark]**

c) If the system must provide a minimum flow of 350 L/s, which pump is better? (i.e., which pump can provide the minimum flow?)

Solution:

If a flow of 350 L/s is required, only the second pump (pump 2) can provide this flow. **[1 mark]**

QUESTION 4 [5 MARKS]

An urban subcatchment has area A = 0.45 ha and time of concentration t_c = 15 min (0.25 h). Complete the following questions:

a) Estimate the rainfall intensity for a 100-year storm (T = 100 years) using the IDF equation $i = \frac{a}{(t_c+b)^c}$, where i = average rainfall intensity (mm/h), a = 59.7 mm/h, b = 0.33 h, and c = 0.8. Note: Be sure to enter t_c into the equation in units of hours!

Solution:

Use the IDF equation: $i = \frac{a}{(t_c+b)^c} = \frac{59.7}{(0.25+0.33)^{0.8}} = 92.3\ mm/h$ **[2 marks]**

b) Use the Rational Method to estimate the peak runoff rate for the subcatchment under the 100-year storm. Use a runoff coefficient of C = 0.75.

Solution:

Rational method equation: $Q = kCiA$, where Q = peak runoff (m³/s), k = 1/360 is a unit conversion constant, C = 0.75 is the runoff coefficient, and A = 0.45 ha is the subcatchment area.

Calc. peak runoff rate: $Q = kCiA = \left(\frac{1}{360}\right)(0.75)(92.3\ mm/h)(0.45\ ha) = 0.0865\ m^3/s$, or 86.5 L/s. **[1 mark]**

c) What is the likelihood that at least one 100 year storm will occur during an n = 100 year period? (I.e., what is the failure probability?)

Solution:

Exceedance probability: $P_e = \frac{1}{T} = \frac{1}{100} = 0.01$ or 1%

Failure probability: $P_f = 1 - (1 - P_e)^n = 1 - (1 - 0.01)^{100} = 0.634$ or 63.4%. This means that there is a 63.4% probability of a 100-year storm occurring one or more times during a 100 year period. **[1 mark]**

d) What are the 5 key assumptions of the rational method?

Solution: **[1 mark for all 5 assumptions, 0.5 marks for 3 assumptions]**

1. Time of concentration reached – runoff from all parts of a subcatchment are contributing to the peak discharge
2. Runoff coefficient is constant and independent of soil conditions, rainfall intensity
3. Constant rainfall intensity
4. Uniformly distributed rainfall
5. Frequency of storm and runoff are the same

SUPPLEMENTAL NOTES – USING THE MOODY DIAGRAM

- **<u>Step 1:</u>** Calculate the Reynolds number: $Re = VD/\nu$, where $\nu = 1 \times 10^{-6}\ m^2/s$. Note that Re is dimensionless (no units), so enter V in [m/s], D in [m], and ν in [m²/s].

- **<u>Step 2:</u>** Calculate the relative roughness e/D. It is dimensionless (no units), so enter e in [m] and D in [m]. Locate e/D on the right vertical axis.

- **<u>Step 3:</u>** Trace out the corresponding curve for e/D and move leftward along the curve. Continue tracing left along the curve until you reach the vertical line for Re.

- **<u>Step 4:</u>** Trace a horizontal line left from the point on the curve towards the friction factor axis. The corresponding value on the friction factor axis is f.